A PHOTOGRAPHIC ASSIGNMENT FOR ABSTRACT ALGEBRA

Gregory S. Warrington
Department of Mathematics & Statistics
University of Vermont
16 Colchester Ave.
Burlington, VT 05401 USA
gwarring@cems.uvm.edu

Abstract: We describe a simple photographic assignment appropriate for an abstract algebra (or other) course. Students take digital pictures around campus of various examples of symmetry. They then classify these pictures according to which of the 17 plane symmetry groups they belong.

Keywords: Photography, symmetry, frieze, abstract algebra.

> *The Lepadidæ, or pedunculated Cirripedes, have been neglected under a systematic point of view, to a degree which I cannot quite understand... [1, pg. xi]*

Each of us benefits every day from the classifications carried out by experts. We are indebted to the pathologists who can tell the difference between an inconsequential tumor and the tumor requiring immediate treatment. We trust our lives to the mycophagist who picks a peck of morels while ignoring the false morels. Every aspect of our lives is influenced by the materials and technology stemming from the periodic table. Yet, for the non-expert in each of these fields, the particulars are dull: The valves of the barnacle *Lepas anatigera* are "smooth, or delicately striated" [1, pg. 73]. That's nice.

A classification that often arises in a first-semester abstract algebra course is that of the 17 wallpaper groups [3, pg. 474]. To a mathematician, this is a wonderful classification. The language of symmetry allows us to make sense of the myriad two-dimensional patterns we see repeated in our world. It is a simple matter to show our students how to identify patterns from the various groups; it is not hard for them to follow a flowchart (see, e.g., [3, pg. 475]). In this way, the students get a feeling for the mechanics of the classification. But it is akin to long division; there is little excitement in finding the answer.

In an abstract algebra course taught in the spring of 2008, I included a photography component in the course. In short, I directed the students to scour the campus looking for repeating two-dimensional patterns. We then classified these "wild" patterns and, in a second round, actively searched for those groups among the 17 which were not already represented. I also asked them to search for examples of rotational symmetry and for any of the seven one-dimensional frieze patterns. In Figure 1 we reproduce two different-looking student pictures that nonetheless have the same symmetry group. Some examples of photographed objects are a manhole cover, fabrics, fences and a necktie (for the wallpaper groups); a drain, a frisbee-golf basket, a lamp and a Ferris wheel (for rotational symmetry); and bike racks, an architectural truss and a crosswalk (for the frieze patterns).

Figure 1: Student pictures depicting representatives of the *pgg* symmetry group. The left picture was taken by Lauren Bergen, the right by Pam Marcott.

This exercise had a number of benefits for the students. First, by moving the source pictures from the textbook to a physical location that exists in their everyday world, it connects the algebra to their lives. They can point out patterns to their friends. And they are more likely to idly notice the vast number of symmetries we incorporate into everyday objects.

Second, for students who like photography or for those struggling in the course, this exercise is a welcome break from pushing around abstract symbols on a piece of paper. A student can also use the assignment to directly look for symmetries in those objects that play an important role in her life — say her sports equipment or musical instrument.

Third, by classifying objects they themselves have found, there is an ownership involved for these students that would not be there when using pictures from a book. The students become more interested in the actual answer.

I was not expecting, as I ultimately did, to enjoy seeing the students' personalities come out in their choices of images. One dichotomy involved scale: Some of the students were looking for symmetry on the sides of buildings, others were focusing on small sections of fabric. Figure 2 shows two other student photographs from the assignment.

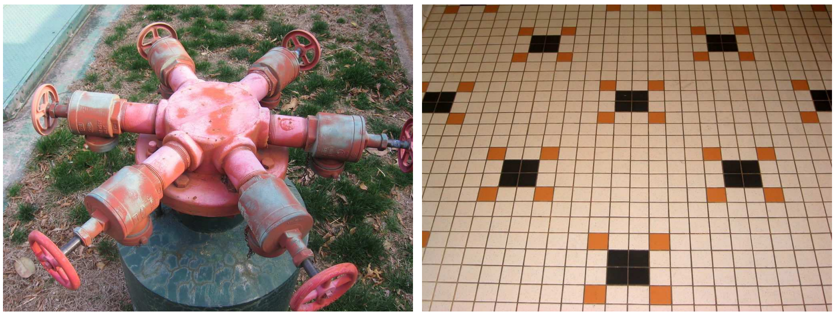

Figure 2: The left picture by Brian Farrell shows an example of D_6 rotational symmetry. The right picture by Beth Allen illustrates the *p4m* symmetry group.

The mechanics of this exercise were surprisingly simple. The vast majority of the students said they had access to a digital camera. My department graciously agreed to lend out the department camera to those who needed to borrow it. I created a *flickr* [2] account for the course to which each student directly uploaded his or her pictures. There was certainly some ambiguity as to which group some of these "real-world" examples should be placed. But this was a feature rather than a drawback. In various ways I had students classify both the photos they had taken and some of those their classmates had taken.

I plan to repeat this activity the next time I teach abstract algebra; the students enjoyed the activity and I did as well. Through repeating this project, I hope to get a better sense of the relative frequency of these various symmetry groups in the world around us. Of course, there is no need to wait for a course. Which of the 17 groups do you think is the most rare?

BIOGRAPHICAL SKETCH

Greg Warrington is an assistant professor of mathematics at the University of Vermont. He received his Ph.D. in mathematics from Harvard University and his B.A. from Princeton University. Greg's research is in algebraic combinatorics. Specifically, he is interested in how concrete combinatorial structures can illuminate underlying algebraic and geometric ideas. He also likes to juggle.

REFERENCES

[1] Darwin, C. R. 1851. A monograph of the sub-class Cirripedia, with figures of all the species. The Lepadidæ; or, pedunculated cirripedes. London: The Ray Society. Volume 1.

[2] http://www.flickr.com.

[3] Gallian, Joseph A. 2006. Contemporary Abstract Algebra. Sixth Edition. Boston: Houghton Mifflin Company.