Juggling performers + Math = ?

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1. Introduction

Everything is mathematical if you look at it from the right perspective. Juggling is too — so long as you are not underneath the flaming torches! But amazingly enough, a mathematical language for juggling was not invented until the early 1980s. Egyptian mathematicians who juggled could not combine their vocations and their avocations. Neither could medieval mathematician/jugglers. But twenty-first century mathematicians can. Consequently, there have been a number of research-level articles concerned with the mathematical *siteswap notation* for juggling. There is even a book, "The Mathematics of Juggling," by Burkard Polster (apparently the movie rights are still available).

We will give a quick sketch of siteswap notation in a moment. However, our goal in this article is something other than simply exploring the beautiful mathematics arising from juggling. Instead, we wish to discover the extent to which jugglers who perform or compete incorporate siteswap notation in their work. If you make your living by juggling on cruise ships, do you pack Polster's book along with your swim suit? If you are about to compete in the Championships of the World Juggling Federation (WJF), do you cram with lists of siteswap tricks, or do you lovingly iron your sequined vest? (Trick question: Clothing with sequins should be hand washed in cold water and left flat to dry.)

To answer these questions we talked to a number of jugglers who perform for the public and/or compete in juggling competitions. It turns out that siteswap tricks appear in both of these arenas, but to different extents.

2. Siteswap notation

Musical notation is a useful analogy to siteswap notation. If you took it upon yourself to invent musical notation, you would have some decisions to make. Should you keep track of the type of instrument on which to play the music, how fast the music should be played, the pitch and duration of the notes, or the color of the cellists socks? For the simplest form of musical notation, the answers are no, no, yes, and no. Pitch and duration are the essence of music. The sheet music is simply the armature an artist uses to build a performance.

For juggling, the essence is the order in which the balls are thrown. To see what we mean, let us consider the two most common three-ball juggling patterns. The first is called the *cascade*. In the cascade, all of the balls are thrown to the same height and to the opposite hand. The balls trace out a lemniscate: ∞. If we label the balls A, B and C, then a record of the throwing order would be ABCABCABC$\cdots$. This string means that ball A is thrown by (say) the right hand, then ball B by the left hand, then C by the right hand, etc. (The notation is ambiguous regarding which hand should make the first throw.) Notice that ball A is not rethrown until balls B and C are thrown.

The second pattern to consider is the *shower*. This is the juggling pattern depicted in cartoons and magazine advertisements for cell phones. (The ubiquity of this pattern in the print media leads some people to conclude that the shower is easier than the cascade. It is not.) In the shower, each ball is thrown to the opposite hand as in the cascade, but the right hand throws the balls high while the left hand throws the balls low. The shape of this pattern is most closely mirrored by an OR gate in logic: ⌂. If we record the throwing order for the shower, we get AABBCCAABBCC⋯.

Jugglers tend to refer to a pattern not by the strings we have used, but by sequences of numbers representing how many beats each ball is in the air. These sequences are easily computed from the letter sequences we have written down by counting the number of spaces between successive appearances of a given letter. For the cascade and shower we get, respectively, $3333333\cdots$ and $151515\cdots$. Furthermore, just as with repeating decimals, we just refer to the shortest repeating subunits: 3 and 51 (or 15). The pattern "n" for any $n > 0$ is technically a siteswap. However, the term siteswap is usually reserved for the more difficult patterns in which there is more than one height to which balls are being thrown.

With this foundation, the mathematician can count juggling patterns, determine which sequences are valid, give a rule for determining how many balls a given pattern requires and take a random walk on a graph generated by siteswap patterns. The mathematical theory is quite elegant. For instance, to determine the number of balls a given pattern requires, we simply take the average. So 51 requires $(5+1)/2 = 3$ balls. And the formula for the number of length-n patterns using b balls is $(b+1)^n$.

3. Performers

So why would a performer risk her health juggling ping pong balls by spitting them energetically out of her up-turned mouth rather than simply wowing the audience by juggling 6738348344? There are several reasons. First, while some patterns are aesthetically pleasing, others are not. The pattern 6738348344 falls into the "not" category. In the diplomatic words of juggler Colin E., many siteswaps have low "visual impact." Not only are such patterns confusing to look at, the audience does not appreciate the skill required to perform them. Second, the simplest five-ball pattern (i.e., $5555\cdots$) takes months to learn. The other five-ball siteswaps are *much* harder. A juggler can spend her time much more effectively learning other types of tricks. Third, which would *you* rather be able to say you could do?

In light of these drawbacks, it is not surprising that Jason Garfield, president of the aforementioned WJF, told me in reference to siteswaps that "for performers they're meaningless." However, other performers I corresponded with are a less dismissive. There *are* patterns of moderate difficulty that audiences appreciate. For example, "Sylvain" on the the rec.juggling newsgroup lists patterns such 97531, 534 and a synchronous pattern notated 6x4,46x. Sylvain also points out that *any* trick can be described, at least in certain aspects, as a siteswap. For instance, the extremely common shower pattern with $(a+1)/2$ balls (a odd) is denoted $a1$ in siteswap notation. All performers use simple siteswaps like the shower or the cascade. The debate is really over the more complicated siteswaps that are difficult to describe without the siteswap language.

Performer Sean Gandini recognizes the above difficulties and has tried to help audiences work through them:

With complex patterns we feel there is an issue of seeability. Like contemporary classical music the more the viewer has seen the more they will appreciate... we actively try and help see patterns which perhaps out of context would be unnoticed... by using unison, colour coding, counterpointing, being in time with music....

In one creative piece (that can currently be viewed on YouTube), his group chants the patterns being juggled. The troupe *Les Objets Volants* also has a piece, "Contrepoint," centered on siteswaps.

It is undoubtedly true that some performers are stopped from performing siteswap tricks by the fact that they do not know the language. But I suspect these are mostly people who have heard of siteswap notation and just have not bothered to learn it as they do not see the benefit.

While siteswaps are having limited success making it onto the stage, many performers juggle siteswaps frequently in their practice for their own enjoyment and to build skill. A number of jugglers echoed Michael Borel's sentiment that

they have definitely made me a better juggler. Siteswaps require accuracy and height control, both of which I have become far more adept at since learning siteswaps.

4. Sports jugglers

The International Jugglers' Association (IJA) has been in existence for sixty years. Its orientation has been towards artistic juggling performers and to hobbyists. Several years ago, the WJF was founded to address the needs to jugglers who view juggling more as a sport than an art. It is "devoted to the promotion and advancement of the sport of juggling." For the past few years, its championships have been aired on ESPN2. A viewing of the advanced balls competition shows that siteswaps are alive and well in the world of sport juggling. Thomas Dietz won this competition in 2006. Jason Garfield told me that "almost everything Thomas Dietz does is a siteswap."

Sports juggling by its very nature is unconcerned with artistic flourishes. A new trick is therefore defined by a few basic components: the locations where objects are thrown and caught, the heights of the throws, and the juggler's body position. For some objects there might be further options. For example, a club is caught by the handle or by the bulb; a ring can by spun like a wheel or flipped like a pancake. Viewed in this light, to avoid siteswaps would be to severely restrict one's repertoire of sports-juggling tricks. One juggler might prefer to incorporate simple siteswaps with his pirouettes and acrobatics while another might stand still while juggling sophisticated siteswaps. Thomas Dietz is apparently the latter type of juggler.

Of course, jugglers do not need to use the siteswap language to create tricks that depend on throwing balls to different heights. In the words of Jason Garfield, siteswap notation is a "more efficient yet geekier way" to refer to such tricks. Some jugglers will, naturally, think and communicate in the language of siteswap more than others will. Regardless, it is interesting to hear the announcers on the ESPN2 airings find a balance between the efficient siteswap language and the clunky English language. The former language is unknown to many people, but the latter did not evolve to contain a nuanced vocabulary for juggling patterns involving throws of different heights.

5. Conclusion

I am a pure mathematician. Personally, I chose this profession because of the aesthetic beauty of the subject and the intellectual satisfaction I get from learning, researching and teaching mathematics. It is always a frustration however, that it is not easier to communicate what I think about to those who are not fluent in language of mathematics.

I think the mathematical language of siteswaps is a microcosm for mathematics as a whole. There is great beauty and utility in what it says about juggling. Mathematics is lending insight. But most juggling performers are primarily concerned with entertaining and impressing lay audiences. Siteswaps do not solve the short-term problems of creating new shows. It is very difficult to convey the mathematical ideas during a traditional juggling performance. Fortunately, some jugglers such as Sean Gandini are trying to do so. In the end, this conveyance should be good both for performers and for audiences. And hopefully, more performers will be convinced that a better understanding of the structure of juggling through siteswaps will help them envision new tricks and entertainments in the long run.

6. Further Reading

The web site www.jugglingdb.com is one-stop shopping for all matters juggling. Look under /compendium/geek for specific information on siteswaps and links to programs that animate siteswap patterns. There are a number of research articles on siteswaps, one by the author is "Juggling Probabilities," *American Mathematical Monthly,* 112, no. 2 (2005) 105–118. The IJA has a web site at www.juggle.org. The WJF can be found at www.thewjf.com. (I am sure that it is a constant source of frustration for the WJF that www.wjf.com is a place to buy cloth baby diapers.) The YouTube clip mentioned above is www.youtube.com/watch?v=2iNdhk2CBqE.

7. Acknowledgments